Multi-Party Indirect Indexing and Applications

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Outline

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Summary

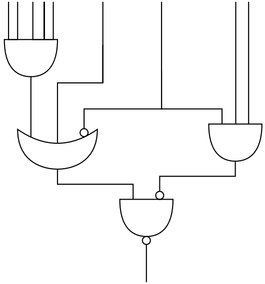
Our main result:

- an efficient multiparty generalization of Naor-Nissim circuits with look-up-tables (LUT)
- a useful (and efficient) generalization of oblivious transfer

Outline

Motivation: using RAM machines in secure MPC

Poly-log reduction of RAM machines to circuits



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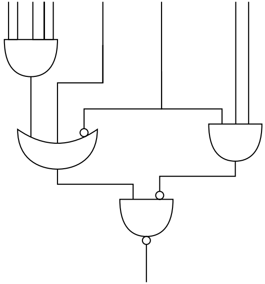

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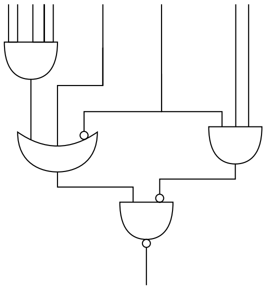

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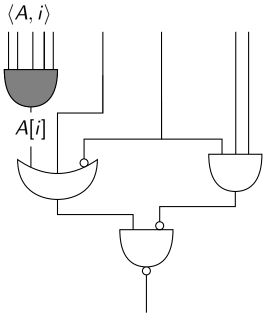

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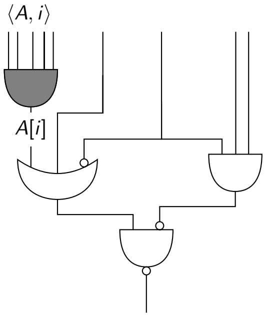
for any RAM machine M running in time $T(n)$ using space $S(n)$, $\exists$ series of circuits with LUT $\{C_n\}_{n\in\mathbb{N}}$ computing f_M, where C_n is size $T(n)\mathrm{polylog}(S(n))$



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Private computation via simulation of a RAM machine appropriate for

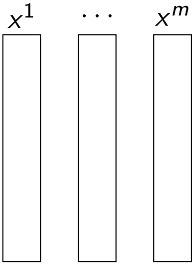
- any problem where a large array of data must be used and
  - only some of the data is ever accessed, or
  - the access pattern leaks information

Example: Private Sampling [IMSW07]

Input

- an m-ary function f
- m inputs of length n,

$$x_j^i = j\text{-th element of } x^i$$



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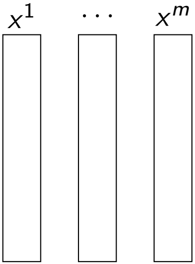
$f(x^1_r, \ldots, x^m_r)$, some secret, random $r \in [n]$



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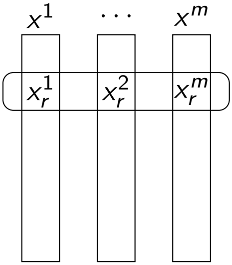

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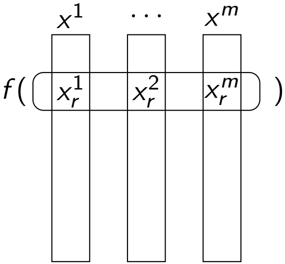

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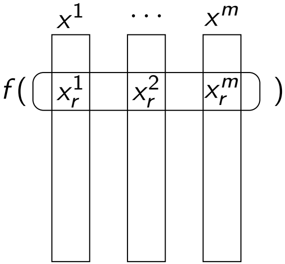


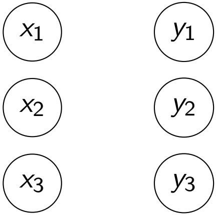
- used in:
  - private approximation (*e.g.* of the sum, of the norm)
  - private data-mining

Example: Private Bipartite Stable Matching [Gol06]

Input

- two sets (men and women) of size n
- a set of rankings

$$x_j^i = k \text{ if } x_i \text{ gives } y_j \text{ rank } k$$



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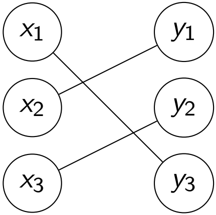

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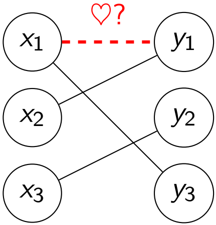

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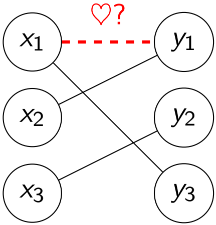

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- *stability*: no unmatched individuals rank one another higher than their "spouse"
- used in:
  - matching residents to hospitals (US, Canada, Scotland)
  - placement of students at universities (Norway, Scotland)
  - professional services (*e.g.* National Matching Services, Inc)

Outline

Introduction: Private mLUT

Input

- Database $\Delta = (\delta_0, \ldots, \delta_{n-1})$
- Party i holds $[\Delta]_i$, a share of Δ
- Party i holds $[\sigma]_i$, a share of secret index σ

Output

- Party i learns $[\delta_\sigma]_i$, a share of $\Delta[\sigma] = \delta_\sigma$

$$\mathrm{mLUT}([\Delta]_1, [\sigma]_1; [\Delta]_2, [\sigma]_2; \ldots; [\Delta]_m, [\sigma]_m) \rightarrow ([\delta_\sigma]_1; [\delta_\sigma]_2; \ldots; [\delta_\sigma]_m)$$

Definition (Private mLUT)

mLUT is t-private if no coalition of up to t parties can learn any information about σ or any of the elements in Δ.

Our Protocol: mLUT

Input

- Database $\Delta = (\delta_0, \ldots, \delta_{n-1})$
- Party i holds $[\Delta]_i$, a share of Δ, where $\bigoplus[\Delta]_i = \Delta$
- Party i holds $[\sigma]_i$, a share of secret index σ

Output

- Party i learns $[\delta_\sigma]_i$, a share of $\Delta[\sigma] = \delta_\sigma$

Protocol

- Let $[\Delta]_i = \Delta^i$
- For $i = 1$ to m:
  - Parties run $\text{g-mOT}(\Delta^i, [\sigma]_i; [\sigma]_{i+1}; \ldots; [\sigma]_{i+m}) \rightarrow ([\delta^i_\sigma]_i; [\delta^i_\sigma]_{i+1}; \ldots; [\delta^i_\sigma]_{i+m})$
- Party i (locally) computes $[\delta_\sigma]_i = \bigoplus[\delta^j_\sigma]_i$.

Outline

Our Subprotocol: g-mOT

Input

- One party holds $\Delta = (\delta_0, \ldots, \delta_{n-1})$
- Party i holds $[\sigma]_i$, a share of σ

Output

Party i holds $[\delta_\sigma]_i$, a share of $\Delta[\sigma]$

$$\text{g-mOT}(\Delta, [\sigma]_1; [\sigma]_2; \ldots; [\sigma]_m) \to ([\delta_\sigma]_1; [\delta_\sigma]_2; \ldots; [\delta_\sigma]_m)$$

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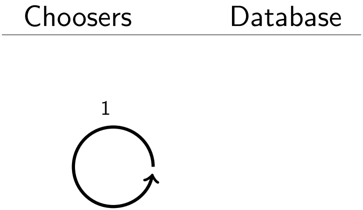
- PIR based on LFAH still efficient when using (traditionally inefficient) special representations of Δ
  - Ex: database as $\log n$-dimensional $2 \times \ldots \times 2$ cube
  - index used by PIR based on binary rep. of index
- (Constant round) protocols exist to convert shares to this form

Our Subprotocol: g-mOT construction (sketch)

1. Choosers create a threshold LFAH encryption system



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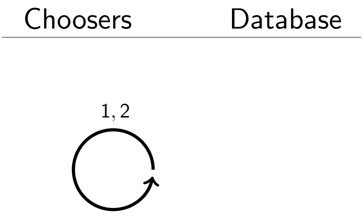

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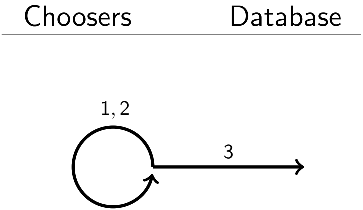

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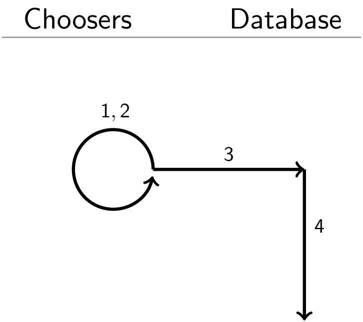

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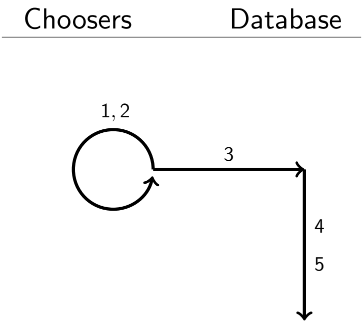

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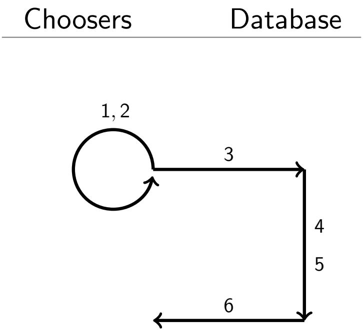

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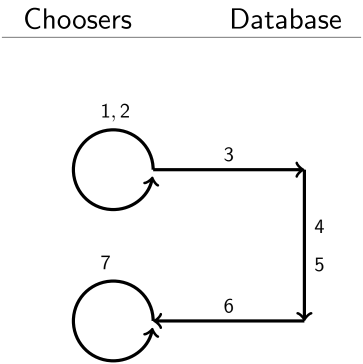

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4. Database uses $E(\sigma)$ to blind Δ
5. Database runs the PIR protocol
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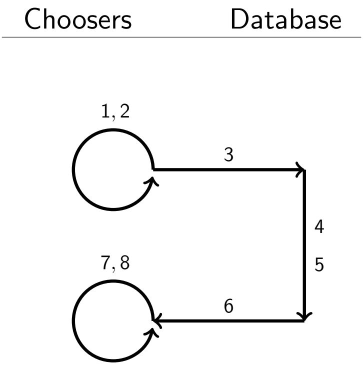
7. Choosers collaborate to decrypt response ($\alpha - 1$ times)
8. Choosers split remaining ciphertext into shares



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- Comm. Complexity:
  - conversion protocol $=$ $b \log b$ multiplications [DFK$^+$06]
  - here, $b = |\sigma| = \log n$
  - multiparty multiplication protocol $= O(m)$
  - thus, total for conversion protocol $= O(m \log n \log \log n)$
  - PIR $= O(k \log^2 n + \ell \log n)$ [Lip03]
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  - Comm: $O(mk \log^2 n + m\ell \log n)$

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  - PIR $= O(k \log^2 n + \ell \log n)$ [Lip03]
- Round:
  - Output share conversion protocols require $O(\log n)$ rounds
  - All other parts of the protocol are constant-round
- Total:
  - Comm: $O(mk \log^2 n + m\ell \log n)$
  - Round: $O(\log n)$

Our subprotocol: Security Claim

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Our protocol is t-private when the Damgård-Jurik LFAH system is IND-CPA secure.

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- in standard model, under Paillier and composite DDH assumptions [DJ03]
- can reduce assumptions: use generic AH system (not LFAH)
- w/ generic AH, round complexity increases by polylog factor
  - less convenient database representation
  - thus more complex share-conversion operations

Outline

Improvements to Private Sampling Applications

Summary

Protocol	Work	Comm.	Round
[IMSW07]	$O(m^2 n)$	$O(m^2 \log n(k \log n + \ell + mk))$	$O(m \log n)$
Ours	$O(m)$	$O(m^2 \log n(k \log n + \ell))$	$O(\log n)$

- for comparison purposes, above ignores costs associated with f
- above, work $=$ number of invocations of the PIR routine by database
- under general AH assumptions, ours remains efficient

Summary

Protocol	Work	Comm.	Round
CT-RSA [FGM07]	$O(n^4\sqrt{\log n})$	$O(mn^3)$	$\widetilde{O}(n^2)$
Ours	$O(n^4)$	$O(mn^2)$	$\widetilde{O}(n^2)$

- in the setting of Golle's private matching algorithm

Thank you.

Thank You.

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Backup: Further Work

- Develop constant-round mOT protocols
- Find natural problems where shared inputs are already in binary representations, for which this work is very efficient (re: round complexity)
- Develop more efficient techniques for oblivious writes
- Find efficient black-box reduction of mOT to 2-party OT
- Consider active adversaries

Backup: LFAH

Message Expansion for α-times encryption

Additive Homo.	$(s+j)k \rightarrow \eta^{\alpha}(s+j)k$
Length-Flexible Additive Homo.	$(s+j\xi)k \rightarrow (s+(j+\alpha)\xi)k$

For [DJ03], $\xi = 1$

Comm. for LFAH-PIR [Lip03]

$$\frac{k\xi}{2}\log^2 n + \frac{3k\xi}{2}\log n + \ell\log n + \ell = \Theta(k\log^2 n + \ell\log n)$$

- database-side comm costs $= (\xi\log n + s)k$
- chooser-side comm costs $= (\frac{\xi}{2}\log^2 n + (s + \frac{\xi}{2})\log n)k$
- where $s = \frac{\ell}{k}$